Module V: Torsion and Twist

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1. Introduction to Torsion

Torsion refers to the **twisting of a structural member** (usually a shaft) when subjected to an **external torque**. The result is shear stress distributed over the cross-section and an angular deformation known as **twist**.

Applicable primarily to **circular shafts** — both solid and hollow — used in mechanical systems like shafts, axles, and helical springs.

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2. Torsional Shear Stress

For a **circular shaft** subjected to torque T, the shear stress at a radius r is:

$$\tau = \frac{T r}{J}$$

Where:

- τ: Shear stress (Pa)
- T: Applied torque (N·m)
- r: Radial distance from center (m)
- J: Polar moment of inertia (m^4)

For a solid circular shaft:

$$J = \frac{\pi d^4}{32}$$

For a hollow circular shaft:

$$J = \frac{\pi (d_o^4 - d_i^4)}{32}$$

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3. Angle of Twist and Torsional Deformation

$$\theta = \frac{T L}{G J}$$

Where:

- θ: Angle of twist (in radians)
- L: Length of shaft

- G: Shear modulus
- J: Polar moment of inertia

This relation helps calculate **twist per unit length** and total angular displacement under torque.

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4. Stepped Shafts

For a shaft with **multiple segments** (different diameters or materials), the total twist is the **sum of individual twists**:

$$\theta_{total} = \sum \left(\frac{T_i L_i}{G_i J_i} \right)$$

Boundary conditions (fixed, free, or loaded ends) must be applied appropriately to determine internal torque in each section.

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5. Torsion in Shafts Fixed at Both Ends

When both ends of the shaft are **fixed** and subjected to external torque(s), compatibility of deformation must be enforced:

- The net angular displacement at the fixed ends is **zero**
- Internal torques are solved using equilibrium and deformation compatibility

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6. Stresses and Deflection in Helical Springs

Helical springs under axial load behave as torsional elements.

a. Shear Stress:

$$\tau = \frac{8 P D}{\pi d^3}$$

Where:

- P: Axial load
- D: Mean coil diameter
- d: Wire diameter

b. Deflection:

$$\delta = \frac{8 P D^3 n}{G d^4}$$

Where:

- n: Number of active coils

Helical springs are widely used for energy absorption and storage.